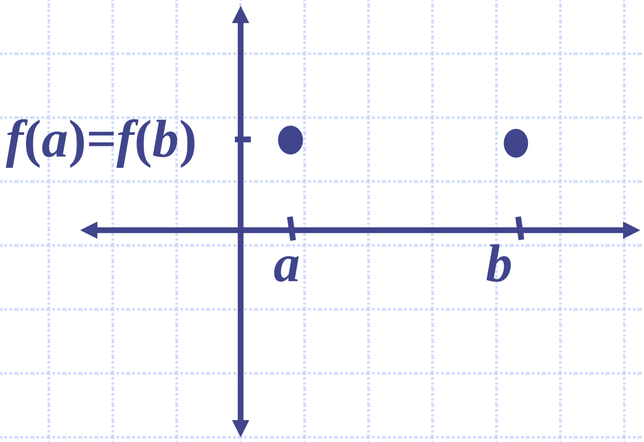
Rolle’s Theorem

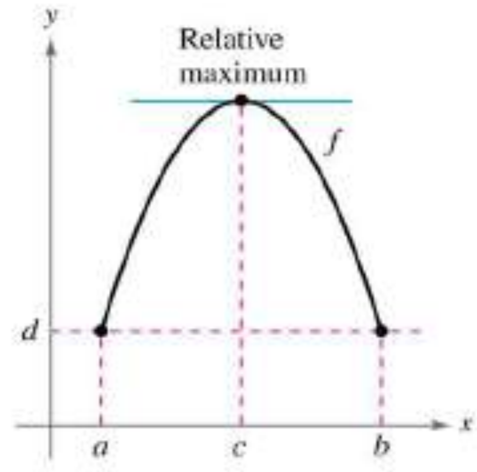
Rolle's Theorem

If you connect from $f(a)$ to $f(b)$ with a smooth curve, there will be at least one place where $f'(c) = 0$

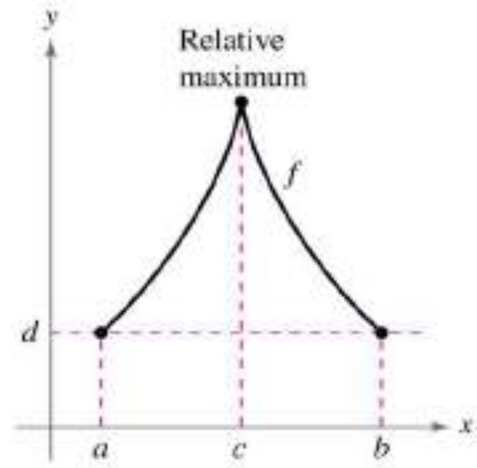


Rolle’s Theorem

Rolle's theorem is an important basic result about differentiable functions. Like many basic results in the calculus it seems very obvious. It just says that between any two points where the graph of the differentiable function $f(x)$ cuts the horizontal line there must be a point where $f'(x) = 0$. The following picture illustrates the theorem.



(a) f is continuous on $[a, b]$ and differentiable on (a, b).



(b) f is continuous on $[a, b]$.

Rolle's Theorem

If two points at the same <u>height</u> are connected by a continuous, differentiable function, then there has to be <u>at least one</u> place between those two points where the derivative, or slope, is <u>zero</u>.

Rolle's Theorem

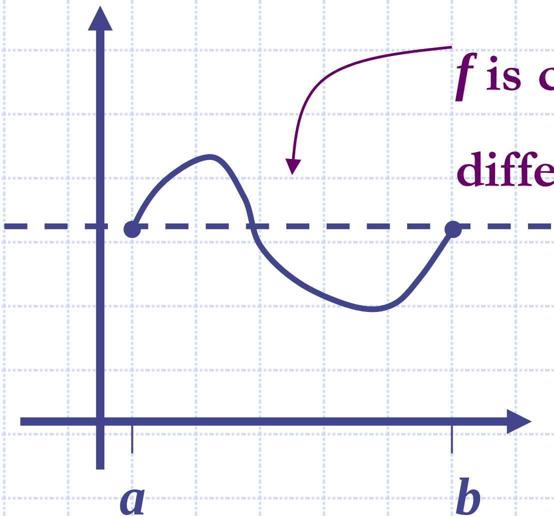
If 1) $f(x)$ is continuous on $[a, b]$,

2) $f(x)$ is differentiable on (a, b), and

3) $f(a) = f(b)$

then there is at least one value of x on (a, b), call it c, such that

$f'(c) = 0$.



Example

Example 1 $f(x) = x^4 - 2x^2$ on $[-2, 2]$

(f is continuous and differentiable)

$$f(-2) = 8 = f(2)$$

Since , then Rolle's Theorem applies…

$$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 0$$

then, $x = -1$, $x = 0$, and $x = 1$

Rolle's Theorem

Does Rolle's Theorem apply?

If not, why not?

If so, find the value of *c*.

Example 2 $f(x) = 4 - x^2$ $[-2, 2]$

Rolle's Theorem

Does Rolle's Theorem apply?

If not, why not?

If so, find the value of *c*.

Example 3 $f(x) = x^3 - x$ $[-1, 1]$

Example

<u>Example 4</u>

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \quad \text{on } [-1, 1]$$

(Graph the function over the interval on your calculator)

continuous on [-1, 1]

<u>not</u> differentiable at 0

<u>not</u> differentiable on (-1, 1)

***f*(-1) = 1 = *f*(1)**

Rolle's Theorem Does NOT apply since

Rolle's Theorem

Does Rolle's Theorem apply?

If not, why not?

If so, find the value of *c*.

Example 5 $$f(x) = \frac{x^2 + 4}{x^2} \qquad [-2, 2]$$

Note

<u>When working with Rolle's make sure you</u>

1. State $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b).
2. Show that $f(a) = f(b)$.
3. State that there exists at least one $x = c$ in (a, b) such that $f'(c) = 0$.

This theorem only guarantees the existence of an extrema in an open interval. It <u>does not tell</u> you <u>how to find</u> them or <u>how many to expect.</u> If YOU can not find such extrema, it does not mean that it can not be found. In most of cases, it is enough to know the existence of such extrema.