Roll No.

Total No. of Pages: 03

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(Sem.-1) B.Tech. Only for CSE/IT (2018 Batch)

MATHEMATICS-I

Subject Code : BTAM-104-18

Paper ID: [75362]

Max. Marks : 60

Time: 3 Hrs.

INSTRUCTIONS TO CANDIDATES :

SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks

SECTION - B & C have FOUR questions each. 2.

Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.

Select atleast TWO questions from SECTION - B & C. 3.

SECTION-A

- 1. Verify Rolle's theorem for $f(x) = x(2-x)e^{\frac{3x}{4}}$ in (0,2)
- Define Beta function and show that it is symmetric.
- Obtain first three terms of Taylor's series of cosx about $x = \frac{\pi}{4}$.

4. If
$$A+B=\begin{bmatrix}1 & -1\\3 & 0\end{bmatrix}$$
 and $A-B=\begin{bmatrix}3 & 1\\1 & 4\end{bmatrix}$ find AB .

- Find rank of the matrix $\begin{bmatrix} 1 & 6 & -2 \\ 2 & 2 & 1 \\ 3 & 8 & -1 \end{bmatrix}$.
- State rank-nullity theorem.
- Define range of a linear transformation.
- Define symmetric matrix, also give suitable example.
- Show that, If zero is an Eigen value of a matrix then it is singular.
- In an n dimensional space every set of 'n + 1' vectors is linearly dependent or independent. Justify your answer.

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SECTION-B

a) Suppose that a function f is differentiate on [0,1] and that its derivative is never zero.
Using mean value theorem, Show that f(0) ≠ f(1)

b) Evaluate the limit
$$\lim_{x \to \frac{\pi}{2}} \left(\frac{1 - sinx}{sinx + \cos 2x} \right)$$
.

- 12. a) Evaluate the integral $\int_{2}^{\infty} \frac{2dx}{x^{2}-x}$, if it exists.
 - b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$ $1 \le x \le 2$ about the x-axis
- 13. a) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ using Gauss Jordan method.
 - b) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$
- 14. Solve the following system of equations by Cramer's rule

$$2x - 2y + z = 1$$
, $x + 2y + 2z = 2$, $2x + y - 2z = 7$

SECTION-C

15. a) By giving proper reasoning determine whether S forms a subspace of Vector space V.

Operations vector addition '+' and scalar multiplications '.' are usual addition and scalar multiplication defined on set of polynomials of degrees less than or equal to $3(P_3)$ and 3-tuple space (V_3) .

If (i)
$$S = \{ p \in P_3 \mid deg(p) = 3 \}, V = P_3$$

(ii)
$$S = \{(x, y, z) | x = 3y\}, V = V_3$$

b) Determine whether the following are Linearly dependent or not?

$$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (1,8,-3)$$

16. a) Let V = P₄, vector space formed by polynomials of degrees less than or equal to 4 under usual addition and scalar multiplication of polynomials. Find the dimension of subspace U of V, where U is

$$S = \{ p \in P_4 \mid p(1) = 0, p'(0) = 0 \}$$

- b) Check whether the transformation T:V₃ → V₂ defined by T(x,y,z) = (x+z, x+y) represent a Linear transformation or not?
- Find the Eigen values and Eigen vectors for the matrix.

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

- 18. a) If A is an orthogonal matrix prove that $|A| = \pm 1$
 - b) Define similar matrices and prove that similar matrices have same eigen values.