Solution

SECTION - A

1 (a). Rolle's theorem :- If a real valued function f is continuous on a proper closed interval $[a, b]$; differentiable in (a, b) and $f(a) = f(b)$ then $\exists\ c \in (a, b)$ such that

$$f'(c) = 0$$

Example: $f(x) = x^2 - 3x + 2$ in $[1, 2]$.

Since $f(x)$ is a polynomial and Every polynomial is continuous $\Rightarrow f(x)$ is continuous in $[1, 2]$.

(ii) $f'(x) = 2x - 3$

$\Rightarrow f'(x)$ Exists in $[1, 2]$.

(iii) $f(1) = 1 - 3 + 2 = 0$

$f(2) = 4 - 6 + 2 = 0 \quad \Rightarrow f(1) = f(2)$

All the properties of Rolle's theorem are satisfied

$\Rightarrow \exists\ c \in (1, 2)$ such that $f'(c) = 0$

$$\Rightarrow \quad 2c - 3 = 0 \quad \Rightarrow \quad c = 3/2 \in (1, 2)$$

1 (b) Since $f(x) = \ln(1+x)$.

Maclaurin series is given by

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots \quad -(1)$$

Here $f(x) = \ln(1+x)$ $\qquad f(0) = \ln(1+0) = \ln(1)$

$f'(x) = \dfrac{1}{1+x}$ $\qquad \Rightarrow f'(0) = 1$

$f''(x) = \dfrac{-1}{(1+x)^2}$ $\qquad \Rightarrow f''(0) = -1$

$f'''(x) = \dfrac{2}{(1+x)^3}$ $\qquad \Rightarrow f'''(0) = 2$

Put in (1)

$$f(x) = 0 + x(1) - \frac{x^2}{2!} + 2\frac{x^3}{3!}$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

Since $\displaystyle \lim_{\substack{x \to 0 \\ y \to 0}} \frac{x^2 y}{x^4 + y^2}$

path: 1 let $(x,y) \to (0,0)$ along the path $y = mx$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \left(\frac{x^2 y}{x^4+y^2}\right) = \lim_{x\to 0} \frac{x^2(mx)}{x^4+(mx)^2}$$

$$\Rightarrow \lim_{x\to 0} \frac{mx^{\cancel{3}}}{\cancel{x^2}(x^2+m^2)} \Rightarrow \lim_{x\to 0} \frac{mx}{x^2+m^2} = 0$$

path 2 Let $(x,y) \to (0,0)$ along the path $y = x^2$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \left(\frac{x^2 y}{x^4+y^2}\right) = \lim_{x\to 0} \frac{x^2 \times x^2}{x^4+x^4}$$

$$\Rightarrow \lim_{x\to 0} \frac{x^4}{2x^4} \Rightarrow \frac{1}{2}$$

Thus limit is not same along two paths

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2} \text{ doesn't Exist.}$$

d) $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4.$

$$\frac{\partial f}{\partial x} = 2x + y + 3 \quad ; \quad \frac{\partial f}{\partial y} = x + 2y - 3$$

for maxima and minima

$$\frac{\partial f}{\partial x} = 0 \quad \text{and} \quad \frac{\partial f}{\partial y} = 0$$

$$\Rightarrow \quad 2x + y + 3 = 0$$
$$x + 2y - 3 = 0$$

$2x+y+3=0 \qquad \times 1$
$x+2y-3=0 \qquad \times 2$

$\Rightarrow \quad 2x+y+3=0$
$\quad \cancel{2x}+4y-6=0$
$\quad - \quad - \quad +$

$-3y+9=0 \quad \Rightarrow \quad y=3$

$2x+y+3=0 \quad \Rightarrow \quad 2x+6=0 \quad \Rightarrow \quad x=-3$

$(-3,3)$ is the critical point

Now $\quad r = \frac{\partial^2 f}{\partial x^2} = 2 > 0$

$s = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(x+2y-3) = 1$

$t = \frac{\partial^2 f}{\partial y^2} = 2$

$rt - s^2 = 4-1 = 3 > 0$ and $r > 0$

$\Rightarrow$ $(-3,3)$ is local minima.

There is no local maxima and no saddle point

e). $\displaystyle\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} dy\,dx\,dz$

$\displaystyle\int_{-1}^{1}\int_{0}^{z}\left[y\right]_{x-z}^{x+z} dx\,dz \Rightarrow \int_{-1}^{1}\int_{0}^{z}(\cancel{x}+z-\cancel{x}+z)\,dx\,dz$

$\displaystyle\int_{-1}^{1}\int_{0}^{z} 2z\,dx\,dz$

$\displaystyle\Rightarrow \int_{-1}^{1}(2zx)_{0}^{z}\,dz \Rightarrow \int_{-1}^{1} 2z^2\,dz \Rightarrow \left[\frac{2z^3}{3}\right]_{-1}^{1}$

$\Rightarrow \frac{2}{3}[1+1] = \frac{4}{3}$

f) Ratio test :- Suppose we have a positive terms series $\sum_{n=1}^{\infty} a_n$. Then by ratio test if

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = l$$

(i) $\sum a_n$ is convergent if $l > 1$

(ii) $\sum a_n$ is divergent if $l < 1$

(iii) The test fails if $l = 1$.

g) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n^2}\right)$ is an alternating series.

Take $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n^2}\right) = \sum_{n=1}^{\infty} (-1)^n a_n$.

where $a_n = \frac{1}{n^2}$

$a_{n+1} = \frac{1}{(n+1)^2}$

$$a_{n+1} - a_n = \frac{1}{(n+1)^2} - \frac{1}{n^2} \Rightarrow \frac{n^2 - n^2 - 1 - 2n}{(n(n+1))^2} < 0$$

$\Rightarrow a_{n+1} < a_n \Rightarrow \sum a_n$ is decreasing series

Also $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n^2} = 0$

By Leibnitz test

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ is convergent.

h) Here $A = \begin{bmatrix} 2 & 6 & 2 \\ 5 & 2 & 1 \\ 9 & 14 & 5 \end{bmatrix}$

$R_1 \to \frac{R_1}{2} \Rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 5 & 2 & 1 \\ 9 & 14 & 5 \end{bmatrix}$

$R_2 \to R_2 - 5R_1$ and $R_3 \to R_3 - 9R_1$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & -13 & -4 \\ 0 & -13 & -4 \end{bmatrix}$$

$R_3 \to R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -13 & -4 \\ 0 & 0 & 0 \end{bmatrix}$

Rank of matrix: No. of non-zero rows = 2.

i) Skew Symmetric matrix: A square matrix A is said to be skew-symmetric if $A = -A^T$

Example:

$$\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$$

j) Cayley Hamilton theorem states that Every square ma satisfies its characteristic Equation.

Let A be a square matrix of order 'n'

It characteristic Equation is

$|A - mI| = 0$ where m is the Eigen Value

Let $|A - mI| = f(m)$

then By Cayley Hamilton theorem

$$f(A) = 0.$$

Q.2 (a) $f(x) = \begin{cases} 3 & x = 0 \\ -x^2+3x+a & 0 < x < 1 \\ mx+b & 1 \le x \le 2 \end{cases}$

We will use the concept that; as function is continuous by LHV

$\Rightarrow \lim_{x \to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0^+} (-x^2+3x+a) = 3.$

$\Rightarrow \boxed{a = 3}$

Illy $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\Rightarrow \lim_{x \to 1^+} (mx+b) = \lim_{x \to 1^-} (-x^2+3x+a)$

$\Rightarrow m+b = -1+3+a$

$\Rightarrow m+b = 2+3$

$\Rightarrow \boxed{m+b = 5}$

Now $f(x)$ is differentiable.

Left handed derivative = Right handed derivative

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

$\Rightarrow \lim_{x \to 1^-} (-2x+3) = \lim_{x \to 1^+} (m)$

$\Rightarrow -2(1)+3 = m \Rightarrow \boxed{m = 1}$

as $b+m = 5$

$b+1 = 5$

$\boxed{b = 4}$

2(b) = $\underset{x\to 0}{Lt}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$ $\infty - \infty$ form.

Here $f(x) = \frac{1}{\sin x}$, $g(x) = \frac{1}{x}$

$$\underset{x\to 0}{Lt}\ (f(x) - g(x)) = \underset{x\to 0}{Lt}\ \frac{\frac{1}{g(x)} - \frac{1}{f(x)}}{\frac{1}{f(x)g(x)}}$$

$$= \underset{x\to 0}{Lt}\ \frac{x - \sin x}{x \sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

Apply L-Hospital rule

$$= \underset{x\to 0}{Lt}\ \frac{1 - \cos x}{x\cos x + \sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \underset{x\to 0}{Lt}\ \frac{\sin x}{-x\sin x + \cos x + \cos x} \Rightarrow \underset{x\to 0}{Lt}\ \frac{\sin x}{-x\sin x + 2\cos x}$$

$$\Rightarrow \frac{0}{0+2} = 0$$

3 (a). $\int_0^3 \frac{1}{(x-1)^{2/3}}\, dx$

Here $\frac{1}{(x-1)^{2/3}}$ becomes ∞ at $x = 1$

we will break the limit as

$$\int_0^3 \frac{dx}{(x-1)^{2/3}} = \int_0^1 \frac{1}{(x-1)^{2/3}}\,dx + \int_1^3 \frac{1}{(x-1)^{2/3}}\,dx$$

$$\Rightarrow \left[\frac{(x-1)^{-2/3+1}}{-2/3+1}\right]_0^1 + \left[\frac{(x-1)^{-2/3+1}}{-\frac{2}{3}+1}\right]_1^3$$

$$\Rightarrow \left[3(x-1)^{1/3}\right]_0^1 + \left[3(x-1)^{1/3}\right]_1^3$$

$$\Rightarrow 3\left[(1-1)^{1/3} - (0-1)^{1/3}\right] + 3\left[(3-1)^{1/3} - (1-1)^{1/3}\right]$$

$$\Rightarrow 3\left[0 - (-1)^{1/3}\right] + 3\left[2^{1/3}\right]$$

$$\Rightarrow 3\left[2^{1/3} - (-1)^{1/3}\right]$$

(b) Surface area about y-axis

$$\Rightarrow \int_0^1 2\pi x \frac{ds}{dy}\,dy.$$

where $\frac{ds}{dy} = \sqrt{1+\left(\frac{dx}{dy}\right)^2}$

$x = 1-y. \Rightarrow \frac{dx}{dy} = -1$

$$\frac{ds}{dy} = \sqrt{1+(-1)^2} = \sqrt{1+1} = \sqrt{2}$$

Surface area $= 2\pi \int_0^1 (1-y)\sqrt{2}\,dy$

$$2\sqrt{2}\pi \left(y - \frac{y^2}{2}\right)_0^1$$

$$\Rightarrow 2\sqrt{2}\pi\left[1-\frac{1}{2}\right] \Rightarrow \sqrt{2}\pi.$$

4(a) $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$

Take $X = \frac{x}{y}$; $Y = \frac{y}{z}$, $Z = \frac{z}{x}$

$u = f(X, Y, Z)$

$$\frac{\partial u}{\partial x} = \frac{\partial f}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial x} + \frac{\partial f}{\partial Z}\frac{\partial Z}{\partial x}$$

$$= \frac{\partial f}{\partial X}\left(\frac{1}{y}\right) + \frac{\partial f}{\partial Y}(0) + \frac{\partial f}{\partial Z}\left(\frac{-z}{x^2}\right)$$

$$= \frac{1}{y}\frac{\partial f}{\partial X} - \frac{z}{x^2}\frac{\partial f}{\partial Z} \quad -(1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial f}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial f}{\partial Y}\left(\frac{\partial Y}{\partial y}\right)$$

$$= \frac{\partial f}{\partial X}\left(\frac{-x}{y^2}\right) + \frac{\partial f}{\partial Y}\left(\frac{1}{z}\right) \quad -(2)$$

$$\frac{\partial u}{\partial z} = \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial z} + \frac{\partial f}{\partial Z}\frac{\partial Z}{\partial z}$$

$$= \frac{\partial f}{\partial Y}\left(\frac{-y}{z^2}\right) + \frac{\partial f}{\partial Z}\left(\frac{1}{x}\right) \quad -(3)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = x\left(\frac{1}{y}\frac{\partial f}{\partial X} - \frac{z}{x^2}\frac{\partial f}{\partial Z}\right)$$
$$+ y\left(\frac{-x}{y^2}\frac{\partial f}{\partial X} + \frac{1}{z}\frac{\partial f}{\partial Y}\right)$$
$$+ z\left(\frac{-y}{z^2}\frac{\partial f}{\partial Y} + \frac{1}{x}\frac{\partial f}{\partial Z}\right)$$

$$\Rightarrow 0$$

b) $z = x\cos y - ye^x$; the point is $(0,0,0)$

tangent plane $(x-x_1)\frac{\partial f}{\partial x} + (y-y_1)\frac{\partial f}{\partial y} + (z-z_1)\frac{\partial f}{\partial z} = 0$

at the point $(0,0,0)$

Let $f(x,y,z) = x\cos y - ye^{x} - z$

$\frac{\partial f}{\partial x} = \cos y - ye^{x}$ at $(0,0,0)$ $\frac{\partial f}{\partial x} = \cos 0 - 0\, e^{0} = 1$

$\frac{\partial f}{\partial y} = -x\sin y - e^{x}$ at $(0,0,0)$ $\frac{\partial f}{\partial y} = -e^{0} = -1$

$\frac{\partial f}{\partial z} = -1$ at $(0,0,0)$ $\frac{\partial f}{\partial z} = -1$

Tangent plane

$$(x-0)\left(\frac{\partial f}{\partial x}\right)_{(0,0,0)} + (y-0)\left(\frac{\partial f}{\partial y}\right)_{(0,0,0)} + (z-0)\left(\frac{\partial f}{\partial z}\right)_{(0,0,0)} = 1$$

$\Rightarrow x(1) + y(-1) + z(-1) = 0$

$\Rightarrow x - y - z = 0$

5(a) area under of the curve $= \int_{x_1}^{x_2} y\,dx$

$\Rightarrow \int_0^{\ln 2} e^{x}\,dx \Rightarrow \left[e^{x}\right]_0^{\log 2}$

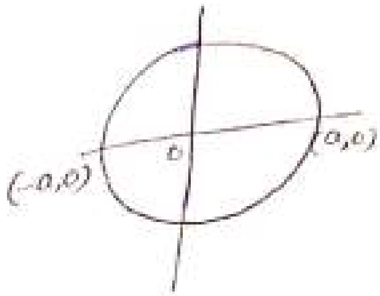
$= e^{\log 2} - e^{0}$

$= 2 - 1 = 1$ square unit

where x^2+y^2

5(b) $\iint (x^2+y^2)\,dx\,dy$

By changing into polar co-ordinates
take $x = r\cos\theta$, $y = r\sin\theta$



$x^2+y^2 = r^2$

$$J\left(\frac{x,y}{r,\theta}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$

$dx\,dy = r\,dr\,d\theta$

$$\int_0^{2\pi}\int_0^{a} (r^2)\, r\, dr\, d\theta \Rightarrow \int_0^{2\pi}\int_0^{a} r^3\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(\frac{r^4}{4}\right)_0^a d\theta$$

$$\Rightarrow \frac{1}{4}\int_0^{2\pi} (a^4)\, d\theta.$$

$$= \frac{1}{4} a^4 (\theta)_0^{2\pi}$$

$$= \frac{1}{4}(2\pi) a^4 \Rightarrow \frac{\pi a^4}{2}$$

SECTION-C

Ans: 6 (a) Let $f(x) = \frac{1}{x^p}$

$f(x)$ is +ve and decreasing from $x \geq 1$.

Cauchy's Integral test is applicable.

Case I If $p = 1$.

$$\int_1^{\infty} f(x)dx = \lim_{t\to\infty} \int_0^{t} \frac{1}{x}\,dx \Rightarrow \lim_{t\to\infty} [\log x]_1^t$$

$$\Rightarrow \log\infty$$

$$\Rightarrow \int_1^{\infty} f(x)\,dx \text{ diverges}$$

$p \neq 1$.

$$\int_1^\infty f(x)dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t x^{-p}dx$$

$$\Rightarrow \lim_{t\to\infty} \left[\frac{x^{1-p}}{1-p}\right]_1^t \Rightarrow \frac{1}{1-p} \lim_{t\to\infty} \left[t^{1-p}-1\right]$$

Subcase I: If $p<1$ then

$$\lim_{t\to\infty} t^{1-p} = \infty \Rightarrow \int_1^\infty f(x)dx \text{ diverges}$$

Subcase II: If $p>1$ then

$$\lim_{t\to\infty} t^{1-p} = \lim_{t\to\infty} \frac{1}{t^{p-1}} = 0$$

$$\int_1^\infty f(x)dx = \frac{1}{1-p}(0-1) = \frac{-1}{p-1}$$

which is finite

$\Rightarrow$ $\int_1^\infty f(x)dx$ converges

$\Rightarrow \sum \frac{1}{n^p}$ converges for $p>1$ and diverges for $p\le 1$.

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7. $a_n = \frac{nx^n}{4^n(n^2+1)}$

Convergence: $a_{n+1} = \frac{(n+1)x^{n+1}}{4^{n+1}((n+1)^2+1)} \Rightarrow \frac{a_n}{a_{n+1}} \Rightarrow \frac{nx^n}{4^n(n^2+1)} \times \frac{4^{n+1}(n..}{(n+1)x}$

$$\frac{a_n}{a_{n+1}} = \frac{n}{n+1} \cdot \frac{4^{n+1}}{4^n} \frac{(n+1)^2+1}{n^2+1} \cdot \frac{x^n}{x^{n+1}}$$

$$\Rightarrow \frac{1}{1+\frac{1}{n}}(4) + \frac{n^2\left[(1+\frac{1}{n^2})^2+\frac{1}{n^2}\right]}{n^2\left(1+\frac{1}{n^2}\right]} \cdot \frac{1}{x}$$

$$\Rightarrow \frac{4}{1+\frac{1}{n}} + \frac{(1+\frac{1}{n^2})^2+\frac{1}{n^2}}{1+\frac{1}{n^2}} \quad \frac{1}{x}$$

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 4 \cdot \frac{1}{x}$$

By ratio test: Σa_n will converge for $\frac{4}{x} > 1 \Rightarrow 4 > x$ or $x <$

Absolutely Convergence: $|a_n| = \left|\frac{nx^n}{4^n(n^2+1)}\right| = \frac{nx^n}{4^n(n^2+1)}$

By the above method we get.

$\Sigma |a_n|$ will converge for $x < 4$

or Σa_n is an absolutely convergent for $x < 4$

Conditionally Convergent: (Convergent but not absolutely convergent)

Σa_n is convergent for $x<4$ and not absolutely convergent for $x >$

Interval of Convergence is $x<4$.

8) $x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2$

$x - y + z = -1$

Solution: Augmented matrix $[A:B]$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5/7 & 20/7 \\ 0 & 0 & -1/7 & -4/7 \end{array}\right] \to$$

$R_4 \to R_4 + 5R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5/7 & 20/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rank of $(A:B)$ = Rank(A) = 3

System is consistent with unique solution

$x + 2y - z = 3$

$-7y + 5z = -8$

$\frac{5}{7}z = \frac{20}{7} \Rightarrow z = 4$

$y = 4$

$x = -1$

9).
$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Characteristic Equation $|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$(8-\lambda)[(7-\lambda)(3-\lambda)-16] + 6\{-6(3-\lambda)+8\} + 2\{24-2(7-\lambda)\} = 0$

$(8-\lambda)\{\lambda^2-10\lambda+5\} + 6\{6\lambda-10\} + 2\{2\lambda+10\} = 0$

$-\lambda^3+18\lambda^2-45\lambda=0$

$\lambda(\lambda^2-18\lambda+45)=0$

$\Rightarrow \lambda = 0, 3, 15$

Eigen Values are 0, 3, 15

The Eigen vector for $\lambda=0$

(i) $(A-0I)(X)=0$.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $$\begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$, $\quad R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad 2x - 4y + 3z = 0$

$-5y + 5z = 0 \quad \Rightarrow y = z$

Take $y = z = 1 \quad \Rightarrow \quad x = \frac{1}{2}$

$X =$ eigen vector $\begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$

for $\lambda = 3$ $\qquad (A - 3I)X = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad R_1 \to R_1 + R_2 \qquad \begin{bmatrix} -1 & -2 & -2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left.\begin{matrix} R_2 \to R_2 - 6R_1 \\ R_3 \to R_3 + 2R_1 \end{matrix}\right\} \qquad \begin{bmatrix} -1 & -2 & -2 \\ 0 & 16 & 8 \\ 0 & -8 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad R_3 \to R_3 + \frac{1}{2} R_2$

$$\begin{bmatrix} -1 & -2 & -2 \\ 0 & 16 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2y + 2z = 0$

$16y + 8z = 0 \quad \Rightarrow \quad z = -2y$ $\qquad X = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

take $y = 1 \quad \Rightarrow \quad z = -2$; $\quad x = -2y - 2z$

$= -2(1) - 2(-2) = 2$

$= 15$

$(A - 15I)X = 0 \Rightarrow \begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1 \to R_1 - R_2$ $\begin{bmatrix} -1 & 2 & 6 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - 6R_1$
$R_3 \to R_3 + 2R_1$ $\begin{bmatrix} -1 & 2 & 6 \\ 0 & -20 & -40 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-x + 2y + 6z = 0$

$-20y - 40z = 0 \Rightarrow y = -2z$

take $z = 1 \Rightarrow y = -2$

$x = 2y + 6z$
$= 2(-2) + 6(1) = 2$

$X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$