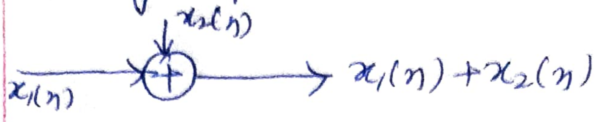
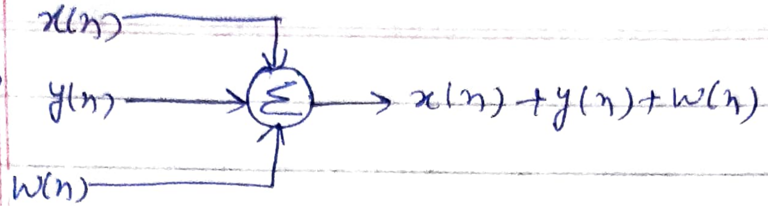
Block Diag Representation

Adder

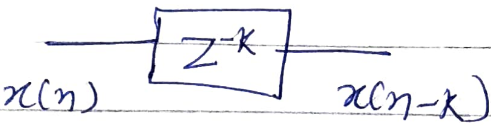


Summing device



Constant Multiplier

Delay by K samples



Advance by K Samples

What is the meaning of canonic and non canonic structures ⇒

If the number of delay elements in the block diagram is equal to the order of difference equation of a digital filter, then the realization structure is called as canonic structure. Otherwise it is called as non-canonic structure

FIR Filters

Transfer function of FIR filter →

$$y(n) = \sum_{K=0}^{M-1} h(K)\, x(n-k) \quad \text{--- (1)}$$

Taking z transform of eq^n (1)

$$Y(Z) = \sum_{K=0}^{M-1} h(K)\, z^{-k}\, X(Z)$$

using time shifting Property

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \sum_{k=0}^{M-1} h(k)\, Z^{-K} \quad \text{--- (2)}$$

Direct form structure →

FIR filter can be obtained by using the eq^n of linear convolution

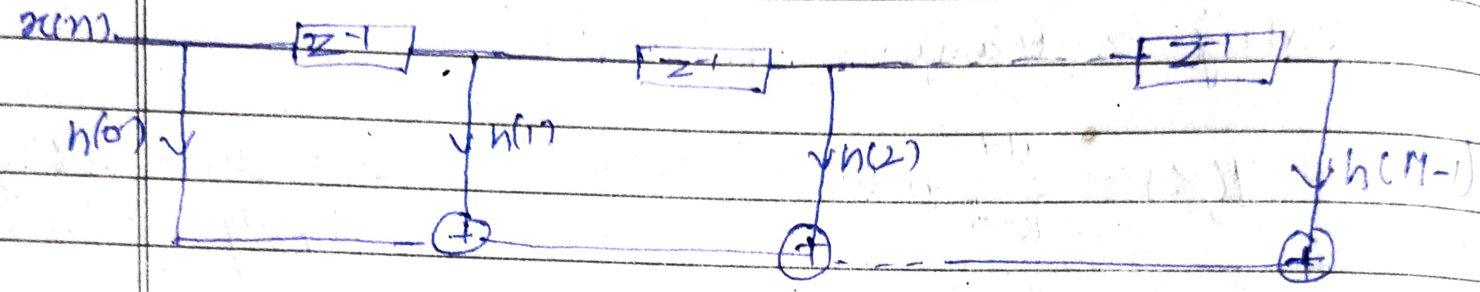
$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K) \quad \text{--- (3)}$$

If we consider that there are M samples in eq^n (3)

$$y(n) = \sum_{K=0}^{M-1} h(K)\, x(n-K) \quad \text{--- (4)}$$

Expand eq^n (4)

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + \cdots$$
$$h(M-1)\,x(n-M+1) \quad \text{---} (5)$$



Cascade Form Structure ⟹

Cascade means the number of stages are connected in series.

$$H(Z) = \sum_{k=0}^{M-1} h(k)\,Z^{-k} \quad \text{---} (6)$$

$$H(Z) = H_1(Z) \cdot H_2(Z) \cdot H_3(Z) \cdots H_k(Z)$$

$H_k(Z)$ is the second order transfer fn

$$H_k(Z) = h(K_0) + h(K_1)Z^{-1} + h(K_2)Z^{-2} \quad \text{---} (7)$$

$$H_k(Z) = \frac{Y_k(Z)}{X_k(Z)}$$

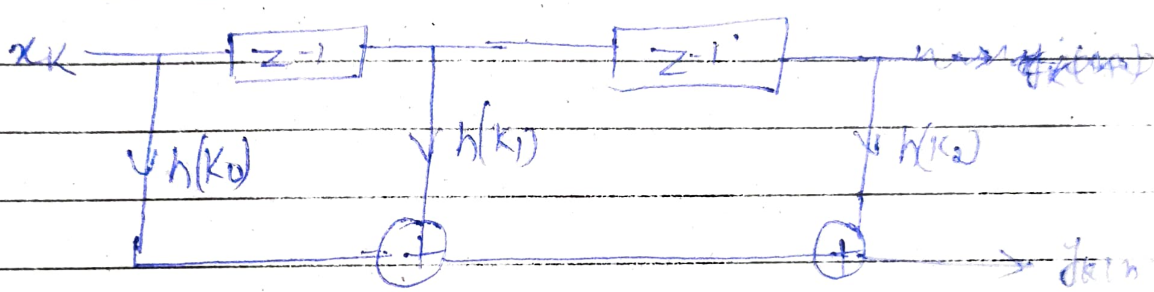
$$\frac{Y_k(Z)}{X_k(Z)} = h(K_0) + h(K_1)Z^{-1} + h(K_2)Z^{-2}$$

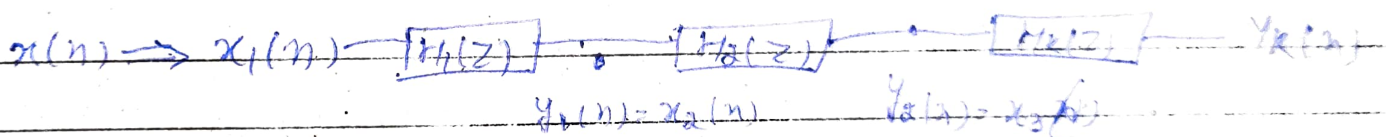
$$Y_K(Z) = h(K_0)\, X_K(Z) + h(K_1)\, Z^{-1}\, X_K(Z) + h(K_2)\, X(Z)\, Z^{-2} \quad \text{--- (8)}$$

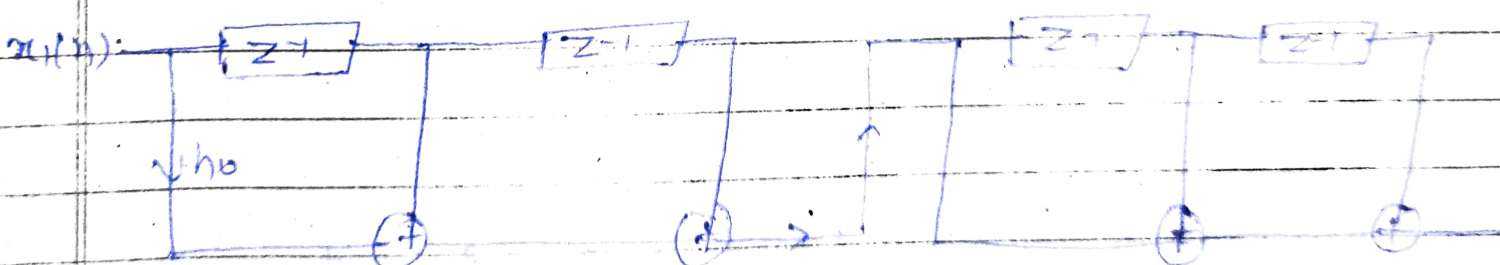
Taking inverse z transform of both sides

$$\cancel{y_x(m) = h(K_0) + h(K_1) z^{-1} + h(K_2) z^{-2}}$$

$$y_K(n) = h(K_0)\, x_K(n) + h(K_1)\, x(n-1) + h(K_2)\, x(n-2) \quad \text{--- (9)}$$







Deduce cascade realization of

$H(Z) = \left(1 + \frac{1}{4}Z^{-1} + Z^{-2}\right)\left(1 + \frac{1}{8}Z^{-1} + Z^{-2}\right)$

For Cascade realization

$$H(Z) = H_1(Z) \cdot H_2(Z)$$

$H_1(Z) = 1 + \frac{1}{4}z^{-1} + z^{-2}$ $\quad H_2(Z) = 1 + \frac{1}{8}Z^{-1} + Z^{-2}$

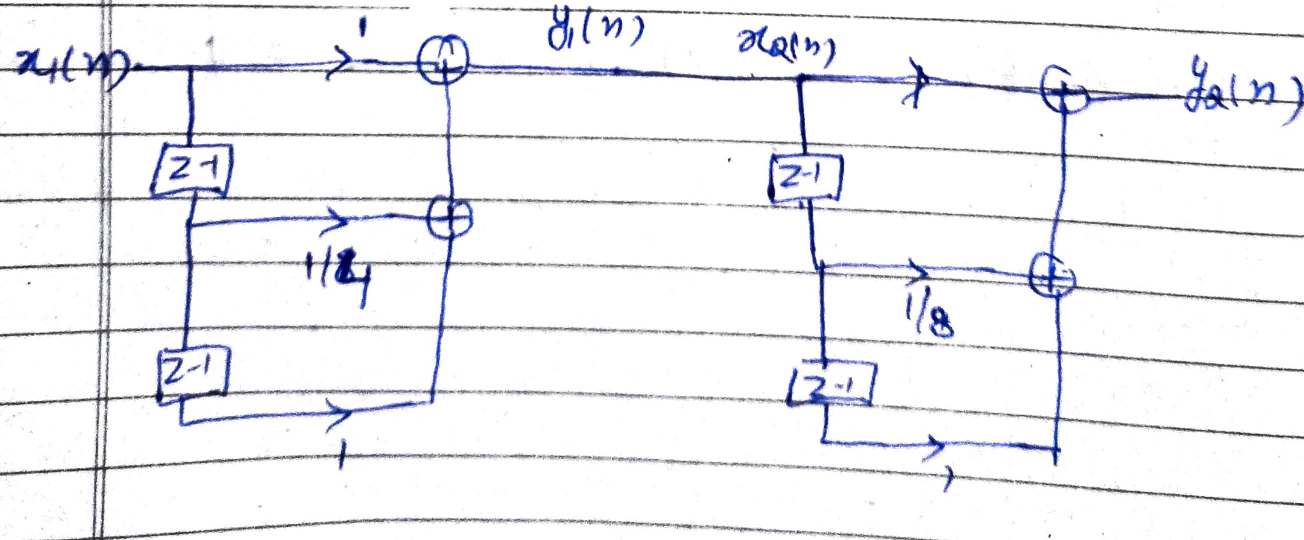
$$H_1(Z) = \frac{Y_1(Z)}{X_1(Z)} = 1 + \frac{1}{4}z^{-1} + z^{-2}$$

$$Y_1(Z) = X_1(Z) + \frac{1}{4}z^{-1}X_1(Z) + z^{-2}X_1(z)$$

Taking IZT

$$y_1(n) = x_1(n) + \frac{1}{4}x(n-1) + x(n-2)$$

$$y_2(n) = x(n) + \frac{1}{8}x(n-1) + x(n-2)$$



Obtain the system for H(Z) and difference eqn for $h(n) = \{1, -2, -2, 3\}$ Draw direct form FIR filter structure

Here range of n is n=0 to 3

$$H(Z) = \sum_{n=0}^{M} h(n) Z^{-n}$$

$$H(Z) = h(0) + h(1) Z^{-1} + h(2) Z^{-2} + h(3) Z^{-3}$$

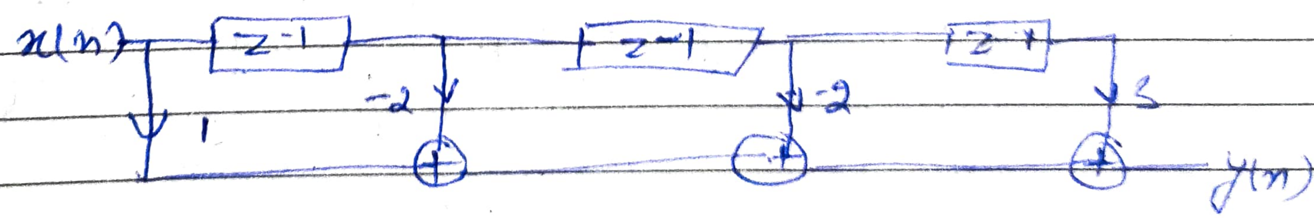
$$H(Z) = 1 - 2Z^{-1} - 2Z^{-2} + 3Z^{-3}$$

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

$$Y(Z) = X(Z) - 2Z^{-1} X(Z) - 2Z^{-2} X(Z) + 3Z^{-3} X(Z)$$

Taking IZT

$$y(n) = x(n) - 2x(n-1) - 2x(n-2) - 3x(n-3)$$



Structures for linear Phase FIR filters ⇒

FIR filter is said to be having a linear phase structure if its unit impulse sequence is either symmetric or antisymmetric about some point in time. That means FIR filter has linear phase if it satisfies the conditions

$$h(n) = h(M-1-n) \qquad n = 0, 1, \ldots M-1 \quad \text{—(1)}$$

Here M = Number of samples

The transfer fun of FIR filter is given by

$$H(Z) = \sum_{n=0}^{M-1} h(n) Z^{-n} \quad \text{—(2)}$$

Let us split the summation into two parts

$$H(Z) = \sum_{n=0}^{\frac{M}{2}-1} h(n) Z^{-n} + \sum_{n=M/2}^{M-1} h(n) Z^{-n} \quad \text{—(3)}$$

But for linear phase we have.

$$h(n) = h(M-1-n) \Rightarrow n = M-1-n$$

Put in second Σ of eqn (3) we get

$$H(Z) = \sum_{n=0}^{\frac{M}{2}-1} h(n) Z^{n} + \sum_{\substack{\cancel{M/2} \\ n=0}}^{\substack{\frac{M}{2}-1 \\ \cancel{M-1}}} h(M-1-n) Z^{-(M-1-n)} \quad \text{—(4)}$$

NOTE The limit of Σ are changes as follow

when $n = M-1-n$

lower limit $n = \frac{M}{2} \Rightarrow M-1-n = \frac{M}{2}$

$$\cancel{\frac{M}{2} - M/1 \neq n} \qquad M-1-\frac{M}{2} = n$$

$$\frac{M}{2} - 1 = n$$

When $n = M-1 \quad = \quad M-1-n = M-1$

$$\cancel{M}-\cancel{1}-\cancel{M}+\cancel{1} = n$$

$$n = 0$$

$$H(Z) = \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[Z^{-n} \cdot Z^{-(M-1-n)}\right] \quad \text{—(5)}$$

Case 1 For even M

We know that $H(Z) = \frac{Y(Z)}{X(Z)}$

$$\frac{Y(Z)}{X(Z)} = \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[Z^{-n} + Z^{-(M-1-n)}\right]$$

$$Y(Z) = \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[Z^{-n} + Z^{-(M-1-n)}\right] X(Z)$$

$$Y(Z) = h(0)\left[1 + Z^{-(M-1)}\right] X(Z) + h(1)\left[Z^{-1} + Z^{-(M-2)}\right] X(Z)$$

$$+ \cdots + h\left(\frac{M}{2}-1\right)\left[Z^{-(\frac{M}{2}-1)} + Z^{-\frac{M}{2}}\right] X(Z)$$

$$Y(Z) = h(0)\left[X(Z) + Z^{-(M-1)} X(Z)\right] + h(1)\left[Z^{-1} X(Z) + Z^{-(M-2)} X(Z)\right] \cdots + h\left(\frac{M}{2}-1\right)\left[Z^{-\frac{M}{2}-1)} X(Z) + Z^{-M/2} X(Z)\right]$$

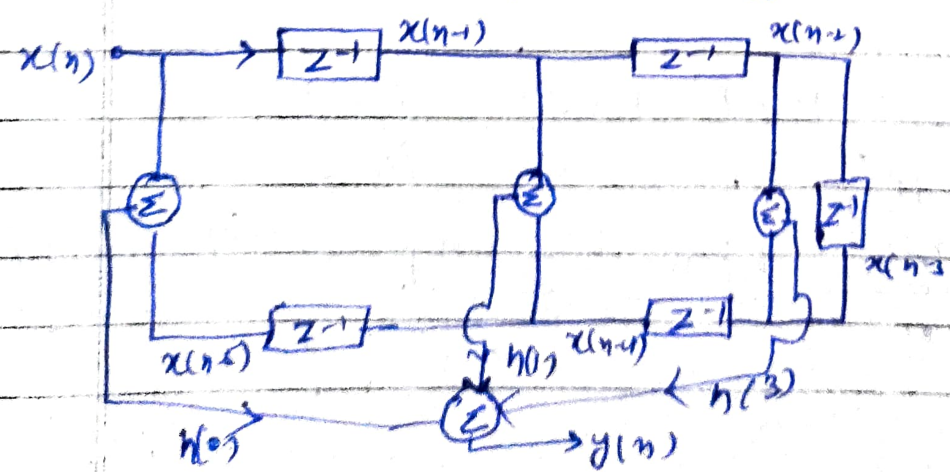
Taking IZT both sides

$$y(n) = h(0)\{x(n) + x(n-(M-1)]\} + h(1)\ x(n-1) + x(n-(M-2))]$$

$$+ \cdots\ h\left(\frac{M}{2}-1\right)\{x(n-(\frac{M}{2}-1) + x(n-\frac{M}{2})$$

Let $M=6$

$$y(n) = h(0)[x(n) + x(n-5)] + h(1)[x(n-1) + x(n-4) + h(2)[x(n-2) + x(n-3)]$$



Case 2 For odd M

When M is odd let $M=5$

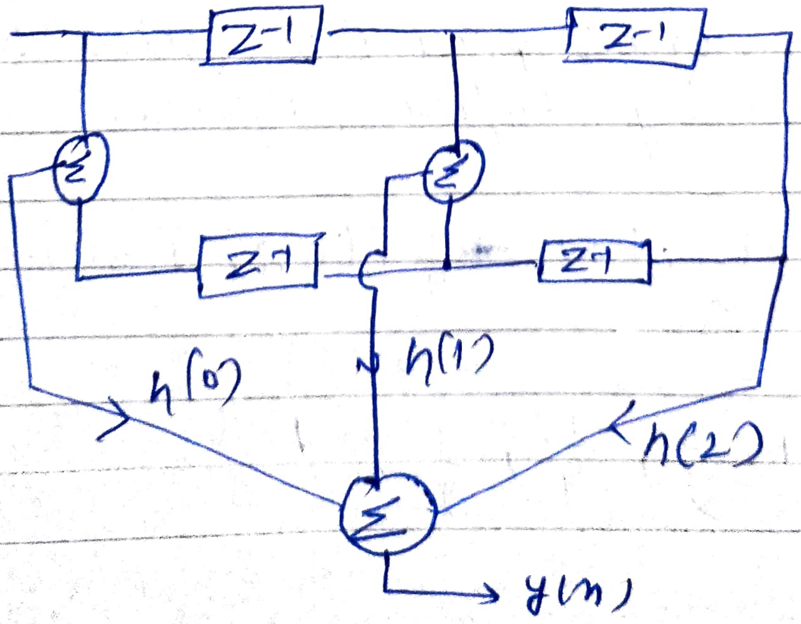
$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2)$
$+ h(3)x(n-3) + h(4)x(n-4)$

But we have the condition for symmetry
$h(n) = h(M-1-n)$

$M=5$

$n=0 \Rightarrow h(0) = h(4)$
$n=1 \Rightarrow h(1) = h(3)$
$n=2 \Rightarrow h(2) = h(2)$
$n=3 \Rightarrow h(3) = h(1)$
$n=4 \Rightarrow h(4) = h(0)$

$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)$
$x(n-3) + h(4)x(n-4)$
$y(n) = h(0)[x(n) + x(n-4)] + h(1)[x(n-1) + x(n-3)]$
$+ h(2)[x(n-2)]$



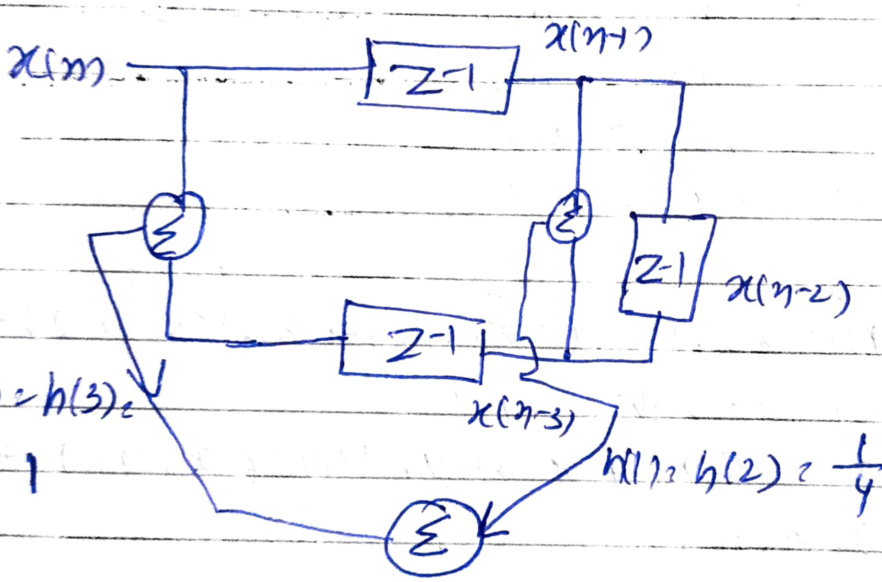
obtain linear Phase realization of H(Z)

$$H(Z) = 1 + \frac{Z^{-1}}{4} + \frac{Z^{-2}}{4} + Z^{-3}$$

$h(0) = 1$, $h(1) = \frac{1}{4}$, $h(2) = \frac{1}{4}$ $h(3)$ 1

$M = 4$ that means even

$h(n) = h(M-1-n)$
$n = (0) = h(0) = h(3)$
$h(1) = h(2)$
$h(2) = h(1)$



Realize a linear phase FIR filter with the following impulse response.

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-3)$$

$$h(n) = \left\{1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{2}, 1\right\}$$

obtain linear Phase realization

$h(0) = 1$, $h(1) = \frac{1}{2}$, $h(2) = -\frac{1}{4}$, $h(3) = \frac{1}{2}$

$h(4) = 1$

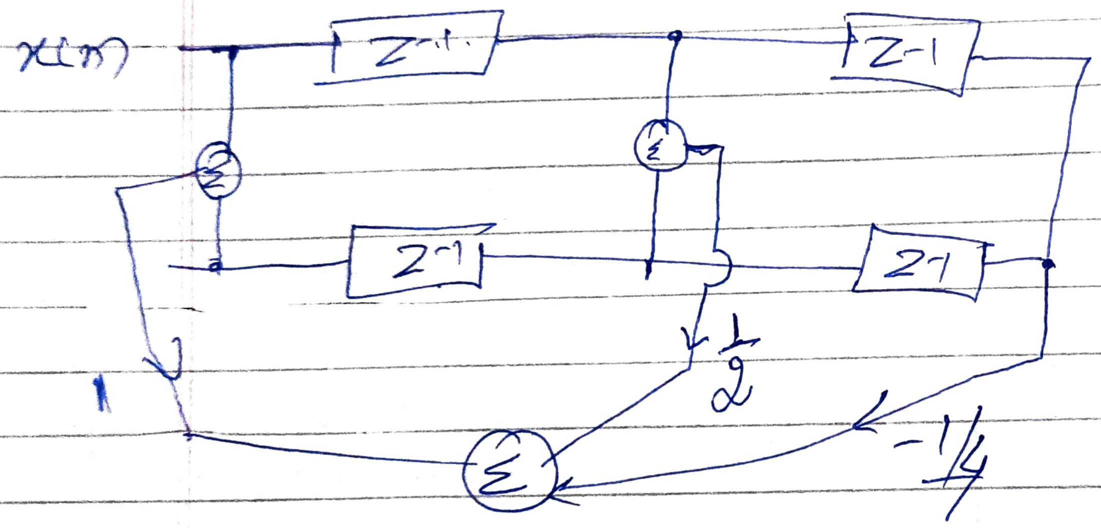
Here $M = 5$

$h(n) = h(M-1-n)$

$n=0 \quad h(0) = h(4)$

$n=1 = h(1) = h(3)$

$n=2 = h(2) = h(2)$



Realize the following system in using minimum no of multipliers

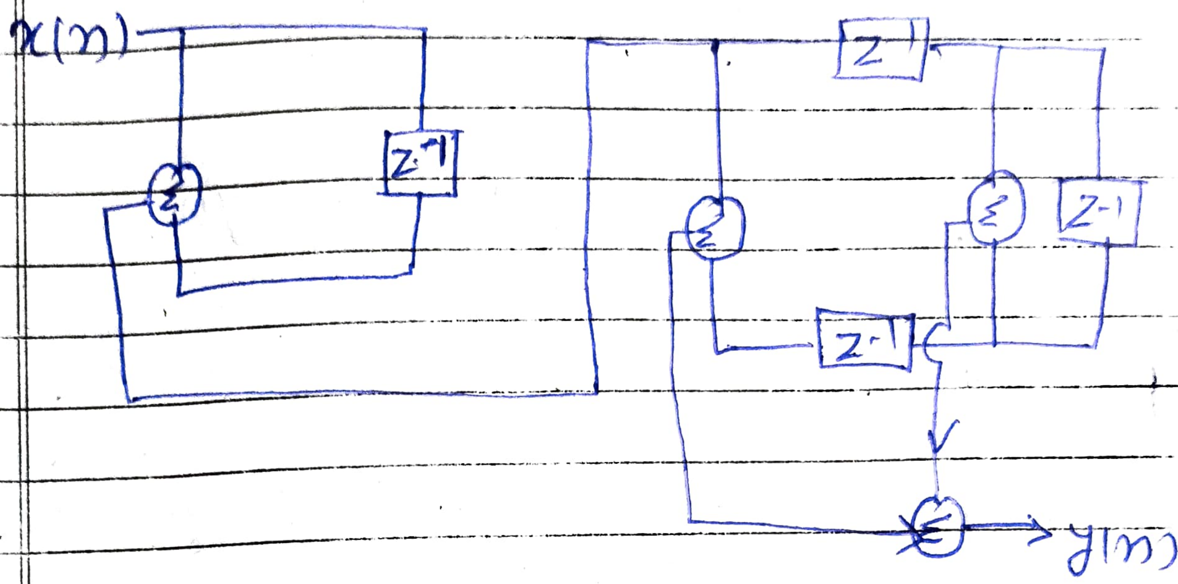
$H(Z) = (1+Z^{-1})\left(1+\frac{1}{2}Z^{-1}+\frac{1}{2}Z^{-2}+Z^{-3}\right)$

$H_1(Z) = 1+Z^{-1}$

$H_2(Z) = 1+\frac{1}{2}Z^{-1}+\frac{1}{2}Z^{-2}+Z^{-3}$

$H_1(z) \Rightarrow h(0) = 1 \quad , h(1) = 1$

$H_2(Z) \Rightarrow h(0) = 1 , h(1) = \frac{1}{2} , h(2) = \frac{1}{2} = h(3) = 1$



Deduce cascade realization of

$$H(Z) = \left(1 + \frac{1}{4} Z^{-1} + Z^{-2}\right)\left(1 + \frac{1}{8} Z^{-1} + Z^{-2}\right)$$

For cascade realization

$$H(Z) = H_1(Z) \cdot H_2(Z)$$

$$H_1(z) = \frac{Y_1(Z)}{X_1(Z)} = 1 + \frac{1}{4} Z^{1} + Z^{-2}$$

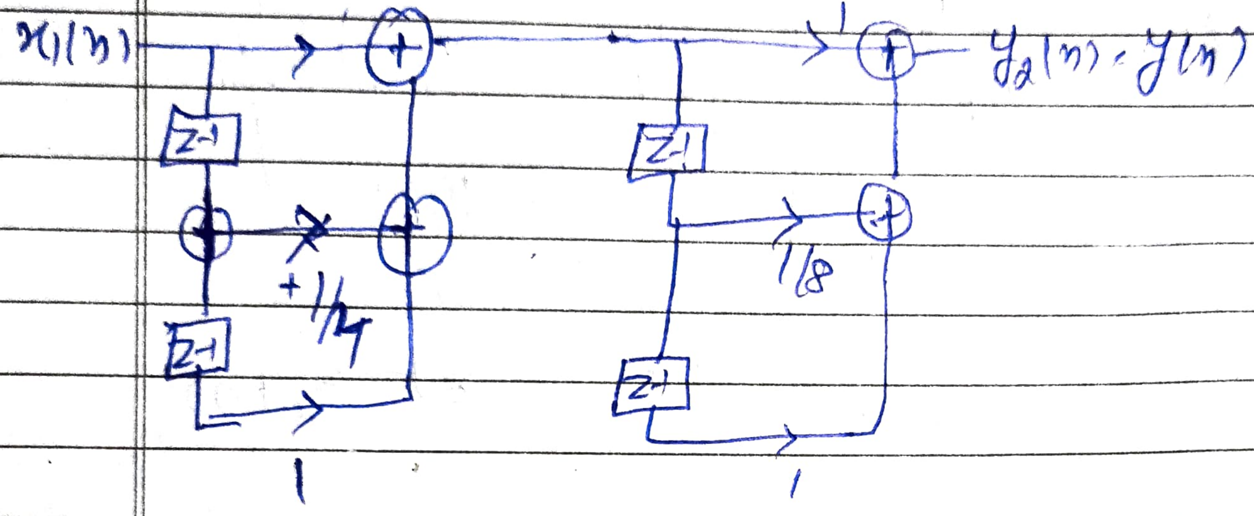
$$Y_1(Z) = X_1(Z) + \frac{1}{4} Z^{1} X(Z) + Z^{-2} X_1(Z)$$

$$Y_2(Z) = X_2(Z) + \frac{1}{8} Z^{1} X_1(Z) + Z^{-2} X_2(Z)$$

Taking IZT we get

$y_1(n) = x_1(n) + \frac{1}{4} x_1(n-1) + x_1(n-2)$

$y_2(n) = x_2(n) + \frac{1}{8} x_2(n-1) + x_2(n-2)$



\# Obtain the System fn H(z) and difference eqn for $h(n) = \{1, -2, -2, 3\}$ Draw direct form FIR filter structure

$H(z) = \{1, -2, -2, 3\}$

range of n is $n = 0$ to 3

using Z transform

$$H(Z) = \sum_{n=0}^{M} h(n) Z^{-n}$$

$h(0) + h(1)Z^{-1} + h(2)Z^{-2} + h(3)Z^{-3}$

$H(Z) = 1 + 2Z^{-1} - 2Z^{-2} + 3Z^{-3}$

Calculation of difference eqn

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = X(z) - 2z^{-1}X(z) - 2z^{-2}X(z) + 3z^{-3}X(z)$$

Taking IZT

$$y(n) = x(n) - 2x(n-1) - 2x(n-2) + 3x(n-3)$$