Fuzzy Propositions

Fuzzy Propositions

- Two-valued logic vs. Multi-valued logic
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Two-valued logic vs. Multi-valued logic

- The basic assumption upon which crisp logic is based - that every proposition is either TRUE or FALSE.
- The classical two-valued logic can be extended to multi-valued logic.
- As an example, three valued logic to denote true(1), false(0) and indeterminacy (1/2).

Two-valued logic vs. Multi-valued logic

Different operations with three-valued logic can be extended as shown in the truth table:

a	b	∧	∨	¬a	⟹	=
0	0	0	0	1	1	1
0	½	0	½	1	1	½
0	1	0	1	1	1	0
½	0	0	½	½	½	½
½	½	½	½	½	½	1
½	1	½	1	½	1	½
1	0	0	1	0	0	0
1	½	½	1	0	½	½
1	1	1	1	0	1	1

Fuzzy connectives used in the above table are:

- AND (∧)
- OR (∨)
- NOT (¬)
- IMPLICATION (⟹) and
- EQUAL (=)

Three-valued logic

Fuzzy connectives defined for such a three-valued logic better can be stated as follows:

Symbol	Connective	Usage	Definition
$\neg$	NOT	$\neg P$	$1 - T(P)$
$\vee$	OR	$P \vee Q$	$max\{T(P), T(Q)\}$
$\wedge$	AND	$P \wedge Q$	$min\{T(P), T(Q)\}$
$\Rightarrow$	IMPLICATION	$(P \Rightarrow Q) or (\neg P \vee Q)$	$\max\{(1 - T(P)), T(Q)\}$
$=$	EQUALITY	$(P = Q) or [(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$	$1 - \lvert T(P) - T(Q) \rvert$

Fuzzy proposition: Example 1

P: Ram is honest

T(P) = 0.0	: Absolutely false
T(P) = 0.2	: Partially false
T(P) = 0.4	: May be false or not false
T(P) = 0.6	: May be true or not true
T(P) = 0.8	: Partially true
T(P) = 1.0	: Absolutely true.

Fuzzy proposition: Example 2

P : Mary is efficient ; T(P) = 0.8
Q : Ram is efficient ; T(Q) = 0.6

- **Mary is not efficient.**

$$T(\neg P) = 1 - T(P) = 0.2$$

- **Mary is efficient and so is Ram.**

$$T(P \wedge Q) = min\{T(P), T(Q)\} = 0.6$$

Fuzzy proposition: Example 2

P : Mary is efficient ; T(P) = 0.8
Q : Ram is efficient ; T(Q) = 0.6

- **Either Mary or Ram is efficient**

$$T(P \vee Q) = max\{T(P), T(Q)\} = 0.8$$

- **If Mary is efficient then so is Ram**

$$T(P \Rightarrow Q) = \max\{1 - T(P), T(Q)\} = 0.6$$

Fuzzy proposition vs. Crisp proposition

- The fundamental difference between crisp (classical) proposition and fuzzy propositions is in the range of their truth values.
- While each classical proposition is required to be either true or false, the truth or falsity of fuzzy proposition is a matter of degree.
- The degree of truth of each fuzzy proposition is expressed by a value in the interval [0,1] both inclusive.

Canonical representation of Fuzzy proposition

- Suppose, X is a universe of discourse of five persons. Intelligent of $x \in X$ is a fuzzy set as defined below.

  Intelligent: $\{(x_1, 0.3), (x_2, 0.4), (x_3, 0.1), (x_4, 0.6), (x_5, 0.9)\}$

- We define a fuzzy proposition as follows:
  P : x is Intelligent
- The canonical form of fuzzy proposition of this type, P is expressed by the sentence $P : v \text{ is } F$.

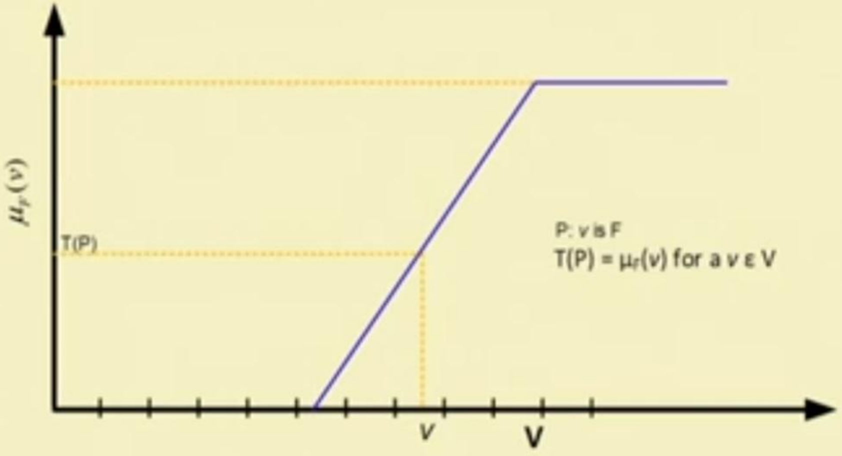
Canonical representation of Fuzzy proposition

- Predicate in terms of fuzzy set.

$P: \; v \; is \; F$; where v is an element that takes values v from some universal set V and F is a fuzzy set on V that represents a fuzzy predicate.

- In other words, given, a particular element v, this element belongs to F with membership grade $\mu_F(v)$.

Graphical interpretation of fuzzy proposition



- ✓ For a given value v of variable V in proposition P, $T(P)$ denotes the degree of truth of proposition P.