SOFT COMPUTING

DEFUZZIFICATION TECHNIQUES

What is defuzzification?

- Defuzzification means the fuzzy to crisp conversion.

Example 1.

Suppose, T_{HIGH} denotes a fuzzy set representing **temperature is High.**

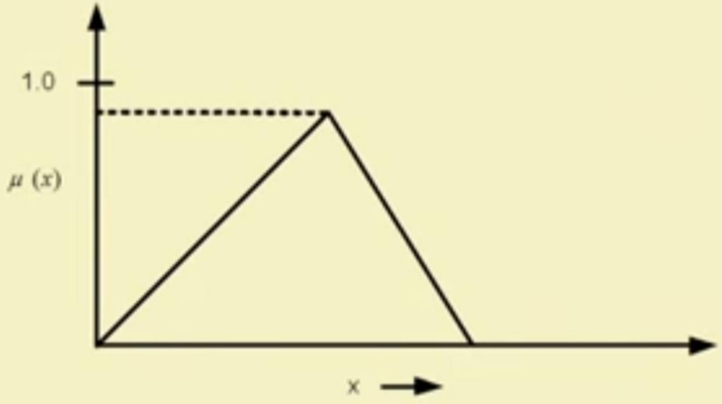
T_{HIGH} is given as follows.

$$T_{HIGH} = \{(15,0.1), (20, 0.4), (25,0.45), (30,0.55), (35,0.65), (40,0.7), (45,0.85), (50,0.9)\}$$

- What is the crisp value that implies the high temperature?

Example 2. Fuzzy to crisp

As an another example, let us consider a fuzzy set whose membership function is shown in the following figure.



What is the crisp value of the fuzzy set in this case?

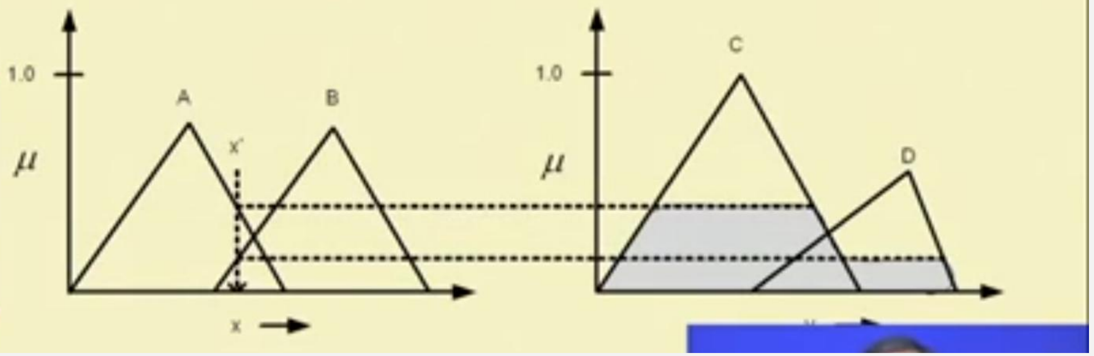
Example 3. Fuzzy to crisp

Now, consider the following two rules in the fuzzy rule base.

$$R1. If\ x\ is\ A\ then\ y\ is\ C$$
$$R2. If\ x\ is\ B\ then\ y\ is\ D$$

A pictorial representation of the above rule base is shown in the following figures.

What is the crisp value that can be inferred from the above rules given an input say x'?



Why defuzzification?

The fuzzy results generated can not be used in an application, where decision has to be taken only on crisp values.

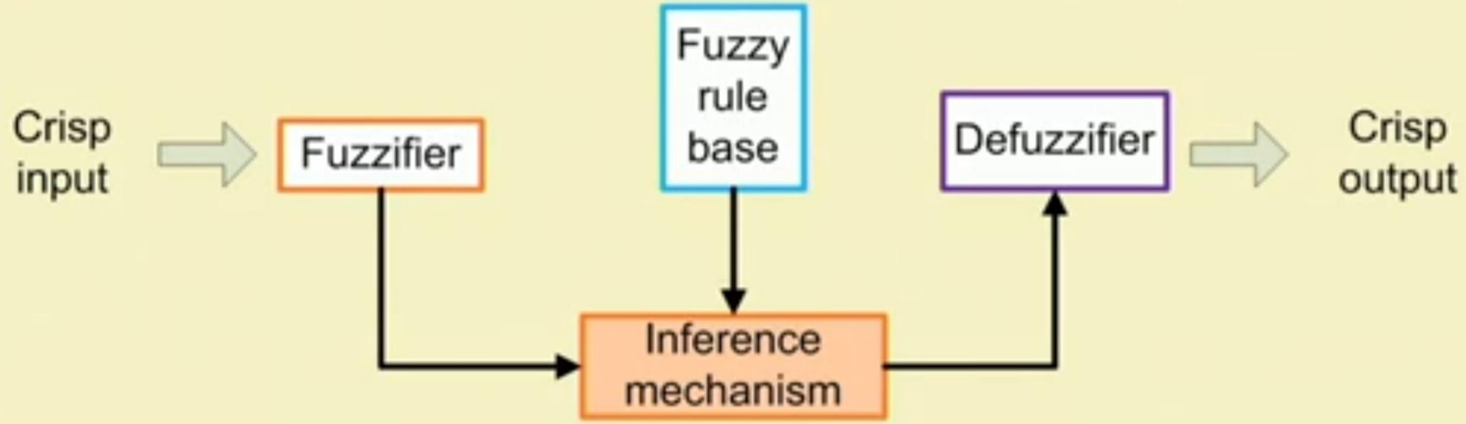
Example.

$$If\ temperature \text{ is } T_{HIGH} \text{ Then } rotation \text{ is } R_{FAST}.$$

Here, may be input T_{HIGH} is fuzzy, but action rotation should be based on the crisp value of R_{FAST} .

Generic structure of a Fuzzy system

Following figure shows a general framework of a fuzzy system.



Defuzzification Techniques

Defuzzification methods

A number of defuzzification methods are known. Such as

1) **Lambda-cut method**
2) **Weighted average method**
3) **Maxima methods**
4) **Centroid methods**

Lambda-cut method

Lambda-cut method is applicable to derive crisp value of a fuzzy set or relation.

- Lambda-cut method for fuzzy relation

The same has been applied to Fuzzy set

- Lambda-cut method for fuzzy set

In many literature, Lambda-cut method is also alternatively termed as Alpha-cut method.

Lamda-cut method for fuzzy set

1) In this method a fuzzy set A is transformed into a crisp set A_λ for a given value of $\lambda (0 \leq \lambda \leq 1)$
2) In other-words, $A_\lambda = \{x | \mu_A(x) \geq \lambda\}$
3) That is, the value of Lambda-cut set A_λ is x, when the membership value corresponding to x is greater than or equal to the specified λ.
4) This Lambda-cut set A_λ is also called alpha-cut set.

Lambda-cut sets : Example

Two fuzzy sets P and Q are defined on *x* as follows.

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

Find the following :

a) $P_{0.2}, Q_{0.3}$

b) $(P \cup Q)_{0.6}$

c) $(P \cup \bar{P})_{0.8}$

d) $(P \cap Q)_{0.4}$

Lambda-cut for a fuzzy relation

The Lambda-cut method for a fuzzy set can also be extended to fuzzy relation also.

Example: For a fuzzy relation R

$$R = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.5 & 0.9 & 0.6 \\ 0.4 & 0.8 & 0.7 \end{bmatrix}$$

We are to find λ-cut relations for the following values of

$$\lambda = 0, 0.2, 0.9, 0.5$$

$R_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $R_{0.2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and

$R_{0.9} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $R_{0.5} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Some properties of λ-cut relations

If R and S are two fuzzy relations, defined with the same fuzzy sets over the same universe of discourses, then

1) $(R \cup S)_\lambda = R_\lambda \cup S_\lambda$

2) $(R \cap S)_\lambda = R_\lambda \cap S_\lambda$

3) $\overline{(R)}_\lambda \neq \overline{R_\lambda}$

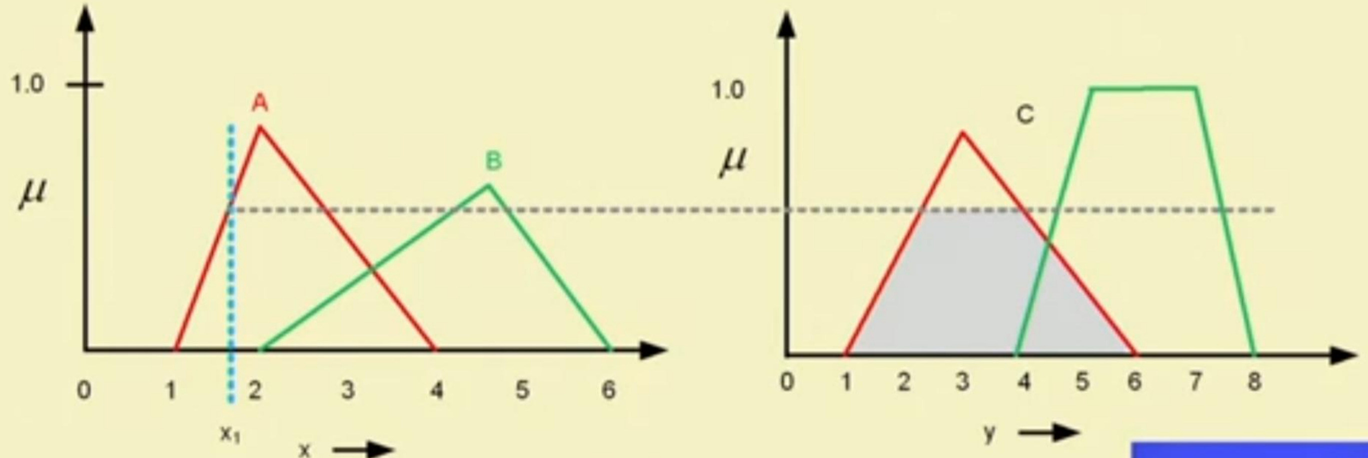
4) For $\lambda \leq \alpha$, where α between 0 and 1, then $R_\alpha \subseteq R_\lambda$

Summary: Lambda-cut methods

Lambda-cut method converts a fuzzy set (or a fuzzy relation) into a crisp set (or relation).

Output fuzzy set : Illustration

The fuzzy output for $x = x_1$ is shown below.



Fuzzy output for x = x_1

Defuzzification Methods

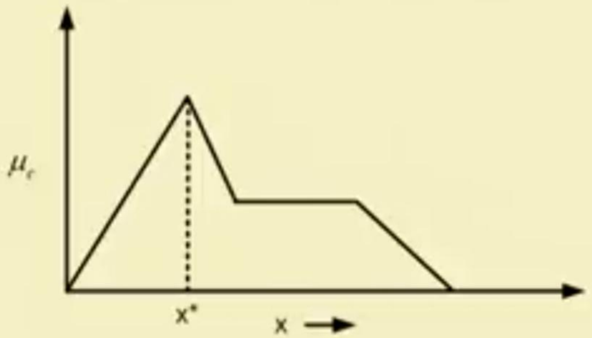
Following defuzzification methods are known to calculate crisp output in the situations as discussed in the last lecture.

1. **Maxima Methods**
   a) Height method
   b) First of maxima (FoM)
   c) Last of maxima (LoM)
   d) Mean of maxima(MoM)
2. **Centroid methods**
   a) Centre of gravity method (CoG)
   b) Centre of sum method (CoS)
   c) Centre of area method (CoA)
3. **Weighted average method**

Maxima method : Height method

This method is based on Max-membership principle, and defined as follows.

$$\mu_c(x^*) \geq \mu_c(x) \text{for all } x \in X$$



Note:

1. Here, x^* is the height of the output fuzzy set C.
2. This method is applicable when height is unique.

MoM : Example 1

Suppose, a fuzzy set **Young** is defined as follows:

$$Young = \{(15,0.5), (20,0.8), (25,0.8), (30,0.5), (35,0.3)\}$$

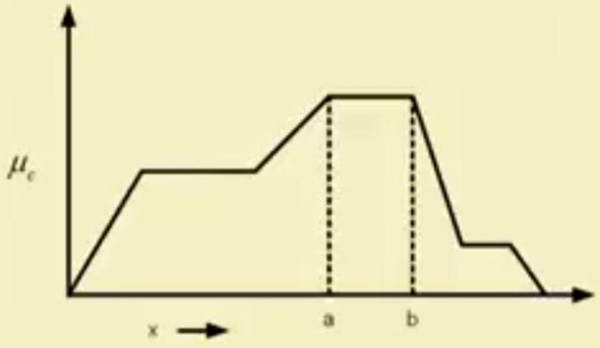
Then the crisp value of **Young** using MoM method is

$$x^* = \frac{20 + 25}{2} = 22.5$$

Thus, a person of 22.5 years old is treated as young!

MoM : Example 2

What is the crisp value of the fuzzy set using MoM in the following case?



$$x^* = \frac{a+b}{2}$$

Note:

- Thus, MoM is also synonymous to middle of maxima.
- MoM is also a general method of Height.

Centroid methods

2. **Centroid methods**

   a) Centre of gravity method (CoG)

   b) Centre of sum method (CoS)

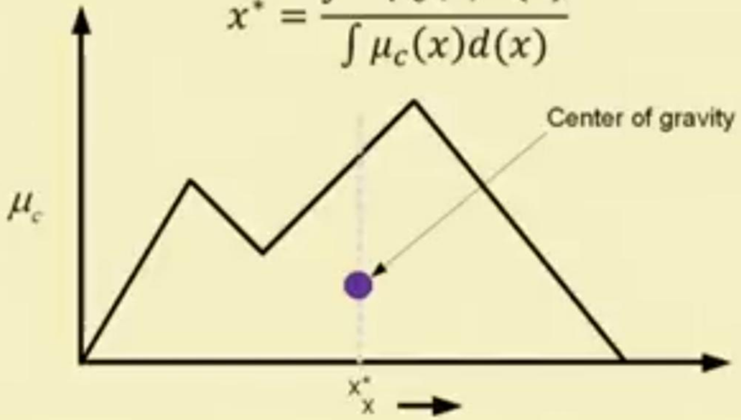
   c) Centre of area method (CoA)

Centroid method : CoG

1) The basic principle in CoG method is to find the point x where a vertical line would slice the aggregate into two equal masses.
2) Mathematically, the CoG can be expressed as follows :

$$x^* = \frac{\int x.\mu_c(x)d(x)}{\int \mu_c(x)d(x)}$$

3) Graphically,



Centroid method : CoG

Note:

1) x^* is the x-coordinate of centre of gravity.

2) $\int \mu_c(x)d(x)$ denotes the area of the region bounded by the curve μ_c

3) If μ_c is defined with a discrete membership function, then CoG can be stated as :

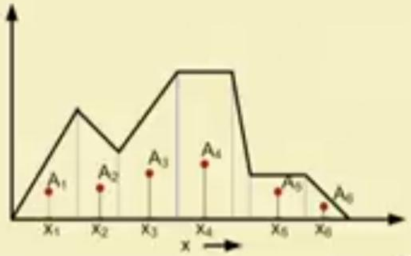
$$x^* = \frac{\sum x_i.\mu_c(x_i)}{\sum \mu_c(x_i)} \quad \text{for } i = 1 \text{ to } n$$

4. Here, x_i is a sample element and n represents the number of samples in fuzzy set C.

CoG : A geometrical method of calculation

Steps:

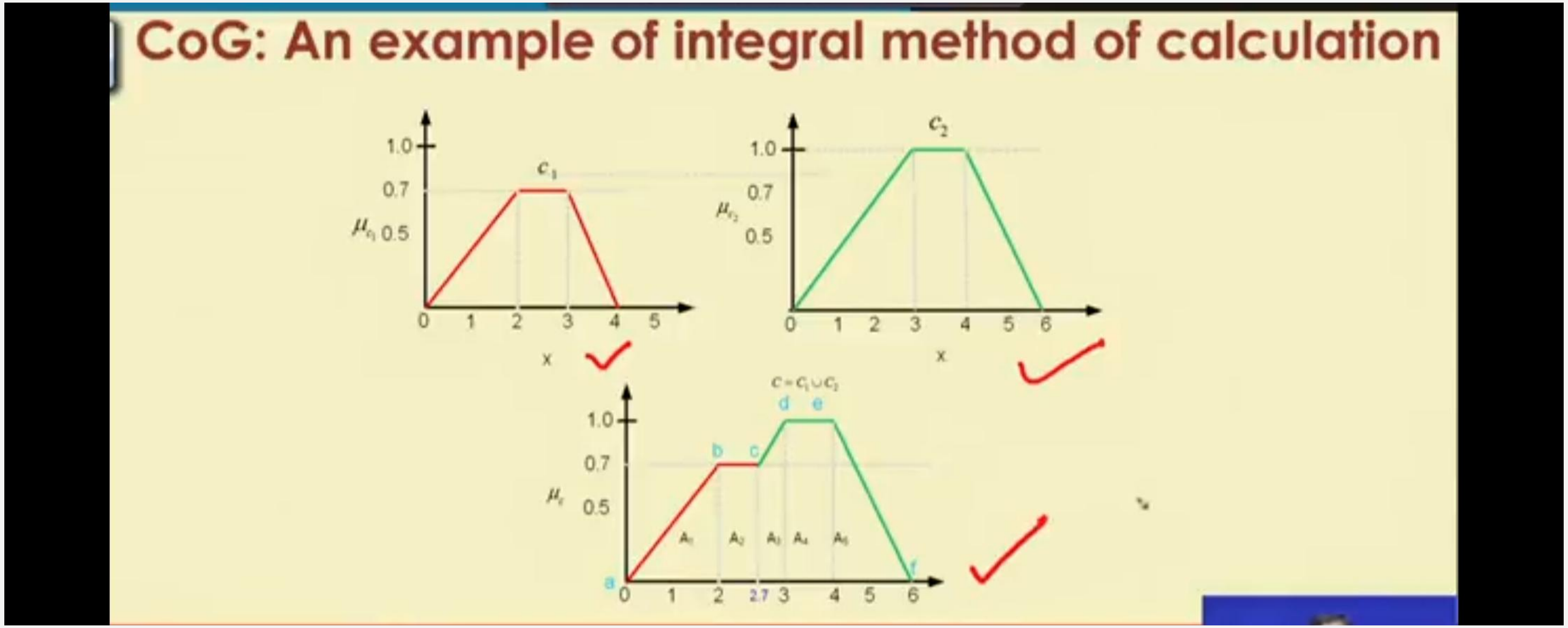
1) Divide the entire region into a number of small regular regions (e.g. triangles, trapezoid, etc.)



2) Let A_i and x_i denotes the area and $c.g.$ of the i^{th} portion.
3) Then x^* according to CoG is

$$x^* = \frac{\sum_{i=1}^{n} x_i.(A_i)}{\sum_{i=1}^{n} A_i}$$

where n is the number of smaller geometrical components.



CoG: An example of integral method of calculation
1.0
0.7
0.5
0 1 2 3 4 5
x
1.0
0.7
0.5
0 1 2 3 4 5 6
x
C = C1 ∪ C2
a b c d e f
A1 A2 A3 A4 A5
0 1 2 2.7 3 4 5 6

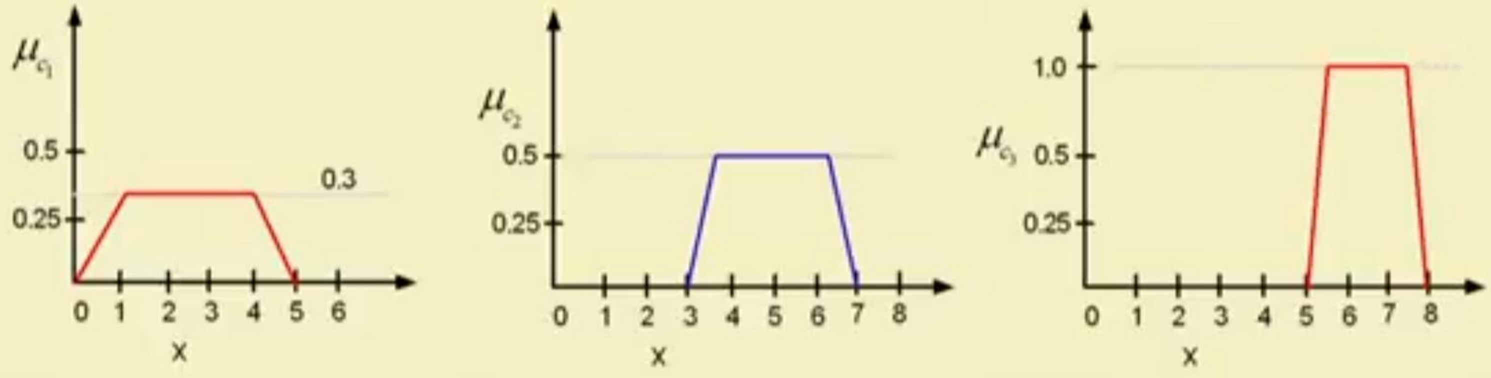
Centroid method : CoS

Note:

- In CoG method, the overlapping area is counted once, whereas, in CoS , the overlapping is counted twice or so.
- In CoS, we use the centre of area and hence, its name instead of centre of gravity as in CoG.

CoS: Example

Consider the three output fuzzy sets as shown in the following plots:



CoS: Example

In this case, we have

$$A_{c_1} = \frac{1}{2} \times 0.3 \times (3+5), x_1 = 2.5$$

$$A_{c_2} = \frac{1}{2} \times 0.5 \times (4+2), x_2 = 5$$

$$A_{c_3} = \frac{1}{2} \times 1.0 \times (3+1), x_3 = 6.5$$

Thus, $x^* = \frac{\frac{1}{2}\times 0.3\times(3+5)\times 2.5+\frac{1}{2}\times 0.5\times(4+2)\times 5+\frac{1}{2}\times 1.0\times(3+1)\times 6.5}{\frac{1}{2}\times 0.3\times(3+5)+\frac{1}{2}\times 0.5\times(4+2)+\frac{1}{2}\times 1.0\times(3+1)}$

Note:

The crisp value of $C = C_1 \cup C_2 \cup C_3$ using CoG method can be found to be calculated as $x^* = 4.9$

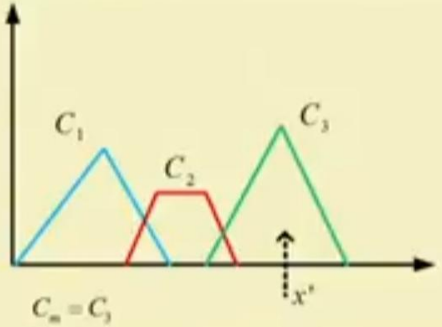
Centroid method: Centre of largest area

If the fuzzy set has two sub regions, then the centre of gravity of the sub region with the largest area can be used to calculate the defuzzified value.

Mathematically, $x^* = \frac{\int \mu_{c_m}(x).x' d(x)}{\int \mu_{c_m}(x)\, d(x)}$;

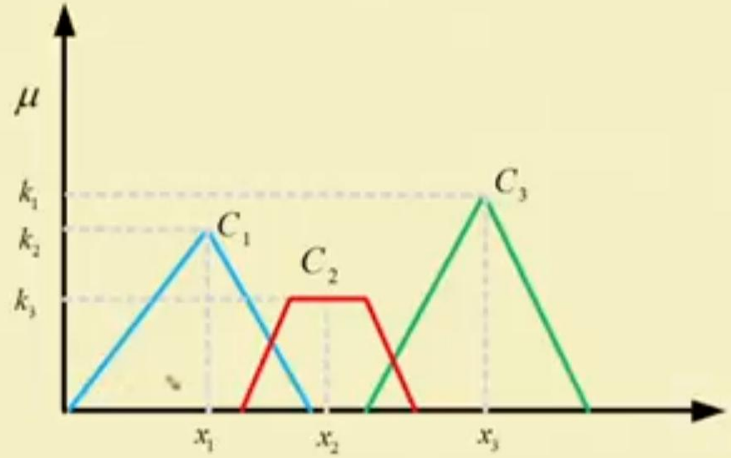
Here, C_m is the region with largest area, x' is the centre of gravity of C_m.

Graphically,

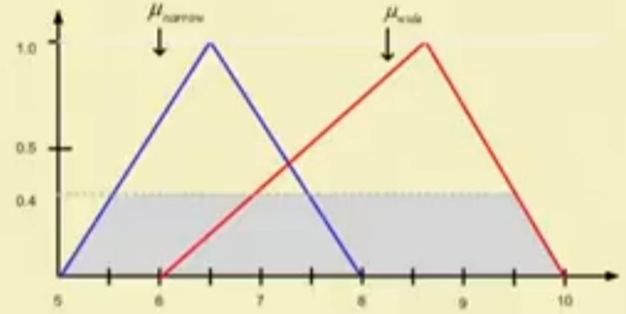


Weighted average method

Graphically ,



Exercise 4



- The width of a road as narrow and wide is defined by two fuzzy sets, whose membership functions are plotted as shown above.
- If a road with its degree of membership value is 0.4 then what will be its width (in crisp) measure.

Hint:

Use CoG method for the shaded region